Renormalized field theory of driven lattice gases under infinitely fast drive

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We use field theoretic renormalization group methods to study the critical behavior of a recently proposed Langevin equation for driven lattice gases under infinitely fast drive. We perform an expansion around the upper critical dimension, $d_c=4$, and obtain the critical exponents to one-loop order. The main features of the two-loop calculation are also outlined. The renormalized theory is shown to exhibit a behavior different from the standard field theory for the driven lattice gas with finite driving, i.e., it is not mean-field-like.

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Since it was introduced by Katz *et al.* [1], the driven lattice gas model (DLG hereafter) has attracted considerable interest [2,3]. Being one of the simplest archetypes of a nonequilibrium model, its study may contribute toward the understanding of out-of-equilibrium systems. The DLG consists of a periodic regular lattice on which nearest-neighbor particle-hole exchanges are performed. The hopping rate is determined by the energetics of the Ising Hamiltonian H, the coupling to a thermal bath at temperature T, and an external uniform driving field $\mathbf{E}$ pointing along a specific lattice axis. In particular, the hopping rate depends on $[(\Delta H+lE)/T]$, where ΔH is the energy variation that would be caused by the configuration change being tried, $E=|\mathbf{E}|$, and $l=1(-1)$ for jumps along (against) $\mathbf{E}$ and 0 otherwise (see [2,3] for a detailed description). The DLG exhibits a continuous phase transition from a disordered state at high T to a stripelike ordered state at sufficiently low T [2,3]. The nature and properties of this transition have been much studied in recent years. A new general Langevin equation has been proposed, designed to capture the physics of the DLG at the critical point [4]. This Langevin equation has different relevant terms for the cases $0<E<\infty$ and $E=\infty$ behaves as a sort of *tricritical point* in the parameter space where some terms are exactly zero and the relevance of the different operators has to be reevaluated. For finite values of the driving field E the Langevin equation previously proposed by Janssen and Schmittmann [6] is recovered. It is the purpose of this paper to investigate the critical behavior of the DLG for $E=\infty$ in order to determine explicitly whether the differences from the $0<E<\infty$ case are relevant and whether, therefore, the critical behavior is changed.

The new Langevin equation reads [4,5]

$$\partial_t\phi=\frac{e_0}{2}\left[-\Delta_\parallel\Delta_\perp\phi-\Delta_\perp^2\phi+\tau\Delta_\perp\phi+\frac{g}{3!}\Delta_\perp\phi^3\right]+\sqrt{e_0}\boldsymbol{\nabla}_\perp\cdot\boldsymbol{\xi}_\perp+\sqrt{e_0/2}\nabla_\parallel\xi_\parallel, \tag{1}$$

where $\nabla_\parallel(\nabla_\perp)$ is the gradient operator in the direction parallel (perpendicular) to the electric field, and the noise satisfies

$$\langle\boldsymbol{\xi}(\mathbf{x},t)\rangle=0,$$
$$\langle\boldsymbol{\nabla}\cdot\boldsymbol{\xi}(\mathbf{x},t)\boldsymbol{\nabla}'\boldsymbol{\xi}(\mathbf{x}',t')\rangle=-\boldsymbol{\nabla}^2\delta(\mathbf{x}-\mathbf{x}')\delta(t-t'). \tag{2}$$

This equation is analogous to a model B in the direction perpendicular to the field (where the energy takes into account the interaction with the parallel direction through the crossed derivatives term), coupled to a simple random diffusion mechanism in the parallel direction. This means that the *relevant ingredient of the infinite driving is the anisotropy it introduces, while the directionality of the flux is irrelevant from a renormalization-group point of view*. A very similar equation has been proposed to describe the *Fréedericksz transition* in nematic liquids, and general asymmetric two-dimensional pattern formation [7].

In order to renormalize this equation, following standard methods [8], let us introduce a Martin-Siggia-Rose response field $\tilde{\phi}$ and recast Eq. (1) as a dynamical functional [9], the associated action of which is

$$\mathcal{L}(\tilde{\phi},\phi)=\int d^dx\,dt\left\{\tilde{\phi}\left[\partial_t-\frac{e_0}{2}(-\Delta_\parallel\Delta_\perp-\Delta_\perp^2+\tau\Delta_\perp)\right]\phi-\frac{e_0}{2}\frac{g}{3!}\tilde{\phi}\Delta_\perp\phi^3-\frac{e_0}{2}\tilde{\phi}(\nabla_\perp^2+\tfrac{1}{2}\nabla_\parallel^2)\tilde{\phi}\right\}. \tag{3}$$

The free propagators are

$$G_{02}^0(\mathbf{k},\omega)=\frac{-e_0(k_\perp^2+\frac{1}{2}k_\parallel^2)}{\omega^2+\left(\frac{e_0}{2}\right)^2k_\perp^4(k^2+\tau)^2},$$
$$G_{11}^0(\mathbf{k},\omega)=\frac{1}{i\omega+\left(\frac{e_0}{2}\right)k_\perp^2(k^2+\tau)}, \tag{4}$$

and the vertex is $-e_0g/12k_\perp^2$. These elements can be represented diagrammatically as in Fig. 1 (wavy legs symbolize response fields; straight lines stand for density fields).

In order to renormalize the theory, one has to look for the primitive divergences in a perturbation expansion. If $\Gamma_{\tilde{n}n}$ denotes a one-particle irreducible vertex function with $\tilde{n}$ exter-

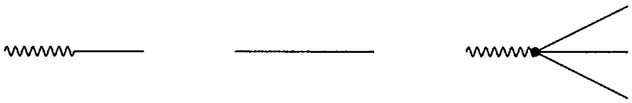

FIG. 1. Elements of perturbation theory: the response and correlation propagators and the four-point vertex. $\tilde{\phi}$ legs are indicated by a wiggly line.

nal $\tilde{\phi}$ legs and n external ϕ legs, only Γ_{11} and Γ_{13} are found to possess primitive divergences. The Feynman diagrams contributing to these vertex functions (shown in Fig. 2) are topologically identical to model B graphs [8]. However, the bare correlation and response propagators that follow from Eq. (3) are anisotropic, in contrast to their counterparts in model B [8].

To one loop in $\varepsilon=4-d$, the ultraviolet divergences in Γ_{11} and Γ_{13} lead to the renormalization of τ and u, the latter being the dimensionless coupling constant $u\equiv A_\varepsilon \tau^{-\varepsilon/2} g$. A_ε is a numerical factor to be defend below. We define renormalized parameters τ_R and u_R by $\tau_R = Z_\tau \tau$ and $u_R = Z_u u$. Given that the leftmost diagram in Fig. 2 does not depend on external moments or frequencies, the derivatives of Γ_{11} with respect to them vanish, and no extra (field) renormalizations are required. The Z factors are determined by the following normalization conditions:

$$\partial_{k_\perp^2} \Gamma_{11}^R|_{\mathrm{NP}} = \frac{e_0}{2}\tau_R ,$$
$$\partial_{k_\perp^2} \Gamma_{13}^R|_{\mathrm{NP}} = \frac{e_0}{2}\tau^{\varepsilon/2} A_\varepsilon^{-1} u_R . \tag{5}$$

A convenient choice for the normalization point NP is $k_i = \omega_i = 0$ and $\tau = \mu^2$, where μ is an arbitrary momentum scale. To one loop, we find

$$\Gamma_{11}(\omega,\mathbf{k}) = i\omega + \frac{e_0}{2} k_\perp^2 (k^2+\tau) + D_1 ,$$
$$\Gamma_{13}(\omega,\mathbf{k}) = \frac{e_0}{2} k_\perp^2 g + D_2 , \tag{6}$$

where D_1 (D_2) corresponds to the algebraic expression of the left (right) diagram in Fig. 2. A calculation in dimensional regularization [10,9] yields

$$D_1 = \frac{3}{64}\frac{g e_0}{7\pi^2} k_\perp^2 \frac{\tau^{1-\varepsilon/2}}{\varepsilon},$$
$$D_2 = -\frac{9}{64}\frac{g^2 e_0}{\pi^2} k_\perp^2 \frac{\tau^{-\varepsilon/2}}{\varepsilon}. \tag{7}$$

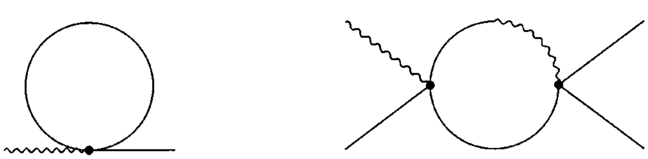

FIG. 2. One-loop diagrams contributing to Γ_{11} (left) and Γ_{13} (right).

After setting $A_\varepsilon = 3/32\pi^2$, one obtains

$$\partial_{k_\perp^2} \Gamma_{11}|_{\mathrm{NP}} = \frac{e_0}{2}\tau\left[1+\frac{u}{\varepsilon}\right],$$
$$\partial_{k_\perp^2} \Gamma_{13}|_{\mathrm{NP}} = \frac{e_0}{2} A_\varepsilon^{-1} \tau^{\varepsilon/2} u \left[1-\frac{3u}{\varepsilon}\right], \tag{8}$$

which entails

$$Z_\tau = 1 + \frac{u}{\varepsilon} + O(u^2),$$
$$Z_u = 1 - \frac{3u}{\varepsilon} + O(u^2). \tag{9}$$

The renormalization-group equation obtained after requiring invariance of the bare irreducible vertex functions upon changes on the normalization point reads

$$[\mu\partial_\mu + \beta\partial_{u_R} + \zeta\partial_{\tau_R}]\Gamma_{\tilde{n}n}^R = 0, \tag{10}$$

where the renormalization-group functions are defined in the usual way: $\beta(u_R) \equiv \mu\partial_\mu u_R$ and $\zeta(u_R) \equiv \mu\partial_\mu(\ln \tau_R)$. A straightforward calculation then leads to

$$\beta(u_R) = -\varepsilon u_R + 3u_R^2 + O(u_R^3),$$
$$\zeta(u_R) = 2 - u_R + O(u_R^2), \tag{11}$$

from which one can determine the location and stability of the fixed points. To this order, apart from the trivial mean-field result $u_R^* = 0$, a nontrivial, infrared stable, fixed point $u_R^* = \varepsilon/3$ emerges. This fixed point controls the critical behavior of the theory below four dimensions.

Now we proceed with the calculation of the associated critical exponents. We first note that, as indicated above, no renormalization of the fields $\tilde{\phi}$, ϕ has been required. Therefore, in particular, the anomalous dimension of ϕ vanishes up to one loop, i.e., $\eta = 0 + O(\varepsilon^2)$. Concerning the exponent $\nu_\perp$, which controls the divergence of the correlation length with temperature [11], we simply have $\nu_\perp = \zeta(u_R^*)^{-1}$, and $\nu_\perp = 1/2 + \varepsilon/12 + O(\varepsilon^2)$. This is to be compared with $\nu = 1/2$, the value obtained by Janssen and Schmittmann in the standard field theory [6]. *This result demonstrates that the continuous version Eq. (1) of the DLG with $E=\infty$ is not mean-field-like but characterizes a universality class other than the one in [6]*. Since there are no dangerous irrelevant operators in Eq. (3) standard scaling laws apply (contrary to the case in [6]). Therefore the exponents are related to each other and estimating η and $\nu_\perp$ is sufficient to deduce all the other exponents. For instance, the order parameter exponent β can be written as $\beta = (\nu_\perp/2)(d-2+\eta)$ [10], and we have $\beta = (1/2) - (\epsilon/6) + O(\varepsilon^2)$.

The previous results concern the one-loop approximation. The two-loop calculation presents an interesting new feature, namely, that the scaling becomes fully anisotropic. In fact, Fig. 3 reveals that, contrary to what happens to one-loop order, the two-loop correction to Γ_{11} depends on external frequencies and momenta; and in the absence of any symmetry between parallel and perpendicular derivatives, one can

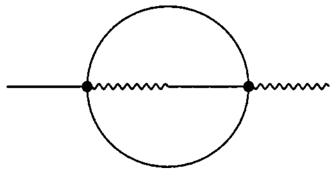

FIG. 3. Two-loop contribution to Γ_{11}.

easily convince oneself by simple inspection that $\partial_{k_\perp^4}\Gamma_{11} \neq \partial_{k_\perp^2}\partial\Gamma_{11}$. In order to absorb these two different divergences, one is constrained to renormalize the parallel and the perpendicular momenta in a different way: if $k_\perp \rightarrow l k_\perp$ then $k_\parallel \rightarrow l^{1+\gamma} k_\parallel$ (where $\gamma \propto \varepsilon^2$ can be determined by explicitly computing the derivatives of the diagram in Fig. 3). The scaling law has to be rewritten as $\beta = \nu_\perp/2(d-2+\gamma+\eta)$, and all the exponents become non-mean-field in this approximation.

Summing up, we have performed the renormalization of the field theory in [4] for the DLG under an infinitely large driving field. The renormalization procedure yields results essentially different from those for a finite field. In particular, corrections to mean field are observed explicitly in the one-loop approximation for the exponent $\nu_\perp$. Anisotropic exponents and a non-mean-field exponent β appear from simple arguments based on the analysis up to two-loop diagrams. These severe differences with respect to the finite driving field case call for extensive computational simulations to observe numerically the physical differences between the two cases.

Note added in proof. We thank Sergio Caracciolo and collaborators for kindly sharing with us unpublished results, which permitted us to detect a small combinatorial error in our original calculation. They have also pointed out to us a problem with infrared singularities in the Langevin equation presented in this paper (see also B. Schittmann *et al.*, e-print cond-mat/9912286). This problem will be tackled in a forthcoming publication.

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[11] We call it $\nu_\perp$ instead of simply ν because the corresponding temperature τ is coupled to $\Delta_\perp$. Note that the temperature in the parallel direction, i.e., coupled to $\Delta_\parallel$, is zero in the bare Lagrangian and does not renormalize. Therefore $\nu_\parallel$ is not defined in our theory.